REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

12[2.05].—J. H. Ahlberg, E. N. Nilson & J. L. Walsh, *The Theory of Splines and Their Applications*, Academic Press, New York, 1967, xi + 284 pp., 24 cm. Price $13.50.

The theory of spline functions has been developed rather recently. However, since Schoenberg's first paper appeared in 1946, the number of research articles devoted to or connected with the theory of splines has grown very fast. The authors have tried to systematize and organize this material and in this reviewer's opinion succeeded rather well.

The simplest spline approximation can be formulated in the following way: Given a function $f(x)$ in the interval $0 \leqq x \leqq 1$. Let $0 = x_1 < x_2 < \cdots < x_n = 1$ be n points. A function $\phi_n(x)$ is a spline approximation if: (1) $\phi_n(x_i) = f(x_i)$, $i = 1, 2, \cdots n$, (2) $\phi_n(x)$ is a polynomial of order three in every subinterval $x_i \leqq x \leqq x_{i+1}$ and (3) $\phi_n(x)$ is as smooth as possible.

Then the following questions arise: Does $\phi_n(x)$ exist for all n ? Does $\phi_n \rightarrow f$ converge for $n \rightarrow \infty$? How fast does ϕ_n converge to f ? All these questions are answered in the first part of the book and other remarkable properties of $\phi_n(x)$ are derived. The rest of the book is devoted to more general spline approximations in one and two space dimensions.

H. O. K.

13[2.05].—E. W. Cheney, *Introduction to Approximation Theory*, McGraw-Hill Book Co., New York, 1966, xii + 259 pp., 24 cm. Price $10.95.

This eminently readable book is intended to be used as a text for a first course in approximation theory. Uniform approximation of functions is emphasized and the discussion is not only theoretical, but provides usable algorithms as well.

An introductory chapter presents some of the major theoretical tools, and assigns an important role to convexity considerations. There follow chapters on the Chebyshev solution of inconsistent linear equations and Chebyshev approximation by polynomials and other linear families. The next chapter treats least-squares approximation and related topics. The scene then shifts back to uniform approximation by rational functions. The final chapter offers a miscellany of topics too interesting to be omitted from the book. Throughout the book the author provides interesting proofs and occasionally new approaches of his own. The approximately 430 problems are an extremely valuable supplement to the text, as is an impressive set of notes to each chapter, which provide the historical context of much of the material and suggestions for further reading. It is not surprising, in view of the great scope of these notes, that there are a few minor misstatements in this part of the book. For example, there is no proof of V. Markoff's theorem in Rogosinski's paper, hence certainly not "the simplest proof". (There is a simple proof of a simpler theorem.) The reader sent to Dickinson's paper for more information about Chebyshev polynomials will not be helped much. These, however, are quibbles in the face of the author's achievement of having written a useful book which is also pleasurable to read.

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14[2.05].—GÜNTER MEINARDUS, *Approximation of Functions: Theory and Numerical Methods*, translated by Larry L. Schumaker, Springer-Verlag, New York Inc., 1967, viii + 198 pp., 24 cm. Price $13.50.

This is a translation of the German edition which appeared in 1964. It differs in detail from that edition by inclusion of new work on comparison theorems for regular Haar systems and on segment approximation.

H. O. K.

15[2.05].—LEOPOLDO NACHBIN, *Elements of Approximation Theory*, D. Van Nostrand Co., Inc., Princeton, N. J., 1967, xii + 119 pp., 20 cm. Price $2.75.

It is appropriate to begin by pointing out that the subject matter of this book is not *best* approximation; the author is rather concerned with the problem of *arbitrarily good* approximation.

More precisely, the author works within the framework of a given function algebra $C(E)$, consisting of all continuous scalar (real or complex, depending on the circumstances) functions on a completely regular topological space E. Such algebras are given the compact-open topology and the general problem is then to characterize the closure of various subsets S of $C(E)$.

The results given include the following cases: S is a lattice (Kakutani-Stone theorem), an ideal, a subalgebra of $C(E)$ (Stone-Weierstrass), or a convex sublattice (Choquet-Deny). In particular, these results imply criteria for the density of S in $C(E)$.

In addition to these well-known theorems, there is a careful presentation of a general *weighted* approximation problem. This problem is a generalization to the $C(E)$ context of the classical Bernstein problem on R^1 or R^N, and is largely based on recent work by the author. The problem is reduced back to the one-dimensional Bernstein problem and various criteria for its solution are then established, making use of analytic or quasi-analytic functions on R^1.

The book will be accessible to readers with a modest background in analysis (Taylor and Fourier series, Stirling's formula) and general topology (partition of unity, Urysohn's lemma). The necessary functional analysis of locally convex spaces is developed in the early chapters. The Denjoy-Carleman theorem on quasi-analytic functions is the only other major result needed and references for its proof are provided. There is an extensive bibliography, but no index or exercises.

It is clear that numerical analysts will find material on approximation more relevant to their profession in, for example, the books of Cheney or Rice. On the other hand, Nachbin's book provides an interesting blend of hard and soft analysis, and more importantly, it collects together for the first time the main closure